Decide Quicker with Total Choice Functions*

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Can we automate the process of decision-making? A possible objective of such a procedure could be to decide quicker and more consistently. As was studied in Decadt et al. [3], it is possible to automate the process of choosing based on previous choices, which are not necessarily binary, in a conservative way. In the same paper a few ways were studied to make the information of the previous decisions more compact, which also makes things more efficient. The problem was stated in the language of choice functions, which is a general mathematical framework that can handle multiple decision rules. To evaluate the choice function, we have used framework of sets of desirable option sets (SODOS's) [2].

We have found that by making the assumption of totality, we can speed up the decision process in the case that the amount of information we are given is large compared to the number of options from which we choose. In particular, we have an algorithm that reduces a decision problem to multiple linear feasibility problems.

Totality is also a reasonable assumption as it is closely related to deciding based on E-admissibility, which also has a practical algorithm that can be found in Utkin and Augustin [4]. Totality assumes that there is a set of total orderings of the options, such that we choose an option if at least one total ordering considers it to be the best option. E-admissibility on the other hand assumes that there is a set of probability measures and we choose an option if there is at least one probability measure that considers it the best one. In De Bock [1] it was proven that choosing based on E-admissibility is equivalent to mixingness and Archimedeanity for the corresponding SODOS. We have proven that totality implies mixingness and we would argue that the difference between totality and mixingness is only subtle in practice. An example where totality gives a different answer than E-admissibility is if we have a choice function C with vacuous information and choose from $\{-u, 0, u\}$, where u is not strictly positive or negative. With totality, we would choose the set $\{-u, u\}$, while with mixingness we cannot reject anything in $\{-u, 0, u\}$.

An interesting approach to SODOS is that we can associate it with classical first-order logic by identifying every option u with a statement "u is preferred over 0". We can interpret the information we are given and our interpretation of coherence as axioms. In this context, some of our results that are similar to a completeness theorem: if the given information is inconsistent, then we can derive it from the axioms and moreover if a statement is true then we can derive it from the axioms. Without totality, the statements "u is preferred over 0" and "$-u$ is preferred over 0" are just mutually exclusive, while with totality the former statement is exactly the negation of the latter one. This makes totality similar to the law of the excluded middle in the context of classical logic. In this frame of mind, we prove that consistency without the law of the excluded middle is the same as when we include it.

References

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