An Accuracy Argument for Coherent Lower Previsions*

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One way to argue that your beliefs are irrational is to show that some other beliefs are better than yours, no matter what. Philosophers have used this strategy to show that, in order for an agent to be rational, her beliefs must be representable by precise probabilities [1]. To set up such an argument we first have to define what it means for some beliefs to be "better" than others. Say we model beliefs by functions mapping propositions to real numbers in $[0,1]$, which represent the agent's confidence in that proposition being true. Then we might say that some such function C' is better than some other C when C' is guaranteed to be more accurate than C, come what may. In this case, we say that C' *dominates* C in accuracy. We can make this more precise by defining an accuracy score $S(C,\omega)$, which measures the accuracy of C at world ω. For a class of such scores it has been shown that, if C is not a probability, there will be some C' that is more accurate than C at every world; whereas if C is a probability, then no such C' exists. If you want to be rational, your beliefs must be probabilistic.

But what if we chose to model beliefs by lower and upper previsions? Since a lower/upper prevision can be any function mapping random variables to the reals, an analogue of the above result would show that coherence is necessary and sufficient to avoid accuracy-domination. This would be particularly appealing for philosophers who believe that rationality norms should be grounded on epistemic considerations (i.e. arguments based on accuracy), as opposed to the pragmatic considerations by which coherence is usually justified for lower previsions. Sadly, a number of impossibility theorems [4] seem to prevent such a result in the imprecise case. They show that, no matter what real-valued accuracy score we pick, it will make some coherent lower prevision dominated in accuracy.

As a response to the impossibility theorems, I follow [2] in proposing that we discuss accuracy-domination with respect to a family of accuracy scores. To see what I mean by this, imagine we have to assess the accuracy of two agents in their beliefs concerning the event H that a tossed coin will come up heads. Agent A has beliefs described by the lower prevision $(\underline{P}(H)=0,\overline{P}(H)=1)$, agent B by $(\underline{P}(H)=1/2,\overline{P}(H)=1/2)$. Which one is more accurate if, say, H is true? The answer must depend on *how we value* certain kinds of accuracy. Think of these lower previsions as sets of probabilities, in the usual way. Then A's set contains the most accurate probability in this case, the one that assigns $P(H)=1$. But it also contains very inaccurate probabilities, including the least accurate one, $P(H)=0$. How *valuable* is it to have accurate probabilities in one's set? How *damning* is it to have inaccurate ones? These are the questions we must answer if we want to compare A's beliefs with those of B.

My approach is guided by the idea that different types of accuracy can be reasonably assigned different values by different agents, or by the same agent in different contexts. This is why I consider a family of accuracy scores $\{S_\alpha : \alpha \in [0,1]\}$, constructed by means of a decision-theoretic analysis like the one in [3]. S_0 does not find it damning at all to have inaccurate probabilities in one's set, and therefore favours the vacuous prevision. S_1 greatly penalises inaccurate probabilities, valuing precise previsions above imprecise ones (indeed, it is a proper scoring rule). And for every $\alpha \in [0,1]$, S_α will model a corresponding mixture of these extreme attitudes. My aim is to show that (a) any incoherent lower prevision will be accuracy-dominated under every $\alpha \in [0,1]$; and that (b) for any coherent lower prevision, there will be some $\alpha \in [0,1]$ that makes it non-dominated. By doing so, I intend to provide a purely epistemic, accuracy-based argument for the coherence of lower previsions over a linear space of random variables.

References

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