Modelling Uncertainty in Quantum Mechanics using Imprecise Probabilities

Keano De Vos KEANO.DEVOS@UGENT.BE
Gert de Cooman GERT.DECOOMAN@UGENT.BE
Jasper De Bock JASPER.DEBOCK@UGENT.BE
Natan T'Joens NATAN.TJOENS@UGENT.BE
Alexander Erreygers ALEXANDER.ERREYGERS@UGENT.BE
Foundations Lab for imprecise probabilities, Ghent University, Technologiepark-Zwijnaarde 125, 9052 Zwijnaarde, Belgium

Quantum mechanics is an advanced theory that models our physical world at the smallest of scales. The theory has held firm for over 100 years, but even though its mathematical framework is clearly established, its interpretation is still open for debate. We use a decision-theoretic approach involving imprecise probabilities to try and get a better grip on one of the essential aspects of quantum mechanics: the inherent uncertainty about the outcome of an experiment. The current framework describes this uncertainty using probabilities, and it models an experiment as follows. A system resides in a certain state, and in order to get information about it, we measure some of its properties, by using a so-called measurement operator. According to quantum mechanics, even when we are sure a system is in a certain state, this does not imply that we can predict the outcome of a measurement with certainty. The possible measurement outcomes are the (real) eigenvalues of the (Hermitian) measurement operator, and Born's rule assigns probabilities to each of these possible eigenvalues. This rule is one of the foundations upon which quantum mechanics is built, but there seems to be no general agreement about where these probabilities come from, and how they should be interpreted. Our aim is to look at this foundational problem from a decision-theoretic point of view, and to show that (a generalisation of) Born's rule can be derived and interpreted using basic axioms, including the usual non-probabilistic axioms of quantum mechanics and simple decision-theoretic principles.

Something along these lines was previously attempted by Deutsch [3]; see also [1] for criticism of his approach. Our argument is similar in spirit, but what it is we want to interpret, and how we go about doing so, is rather different. We see measurements as ways of getting more information about the unknown physical state X of the system. In particular, we see each measurement A as an act (or decision) that produces a certain utility $u_A(X)$ that depends on the system state X. The underlying idea is that a subject's beliefs about what the actual state is, leads them to beliefs about the resulting utility. In particular, based on these beliefs about X, a subject may prefer to perform experiment A with utility function u_A over an experiment B with utility function u_B, because the former is 'expected to' lead to a higher utility. In this way, beliefs about X can be expressed through partial preference orders on measurements—or rather, their utility functions.

We use a number of basic axioms, including the non-probabilistic axioms of quantum mechanics, to derive the form of these utility functions, which we then employ in a decision-theoretic model for expressing preferences between the corresponding measurements/decisions/acts, based on the theory of coherent sets of desirable gambles. Interestingly, our utility functions are closely related—yet not equal—to the gambles that Benavoli et al. [2] use to develop their account of quantum mechanics based on coherent sets of desirable gambles; this allows us to reuse some of their mathematical machinery. They, however, start from different assumptions, and take the form of the gambles as given. We, in contrast, retrieve the form of the utility functions from basic axioms. Nevertheless, their results allow us to arrive at a representation for the uncertainty about X in terms of (sets of) density matrices, and a decision-theoretic justification for (a generalisation of) Born's rule. In this way, we believe to have taken the decision-theoretic approach to explaining the probabilistic aspects of quantum mechanics a significant step further.

References

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