Elicitation of Preference Systems: Two Procedures and their Application to Decision Making under Severe Uncertainty

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In our recent paper Jansen et al. [1] (which is based on our ISIPTA '17 contribution), we introduced a framework for decision making under imprecise probabilities and (potentially very) weakly structured preferences. Specifically, the decision maker's preferences on the consequence set A (i.e. the set in which the considered acts take their values) are modelled by a pair of binary relations (R_1, R_2):

- The (potentially incomplete) relation $R_1 \subseteq A \times A$ encodes the ordinal part of the decision maker's preferences by directly ranking the consequences from the set A themselves (without any notion of strength of preference).
- The (potentially incomplete) relation $R_2 \subseteq R_1 \times R_1$ encodes the cardinal part of the decision maker's preferences by ranking pairs of alternatives $(a,b),(c,d) \in R_1$. Here, $((a,b),(c,d)) \in R_2$ is interpreted as exchanging b by a being at least as desirable as exchanging d by c.

Within this context, the triplet $\mathscr{A} = [A, R_1, R_2]$ is then called the *preference system* of the decision maker and can be viewed as a non-complete version of an algebraic difference structure as discussed, e.g., in Krantz et al. [2, Chapter 4].

Based on this framework, we now present elicitation procedures that enable decision makers to reveal their underlying preference system while having to answer as few as possible simple ranking questions concerning the elements of A. Two different approaches are followed. The first approach directly utilizes the collected ranking data to construct the ordinal part R_1 of the preferences, while the cardinal part R_2 is built up implicitly by collecting meta data on the decision maker's consideration times (i.e. the time the decision maker needs to rank two presented consequences a_i and a_j). The main intuition behind this approach is that ranking a_i and a_j takes less time if the preference of one over the other is of strong intensity. We give exact conditions on the relation between preference strength and consideration time for our method to produce the decision maker's true preference system.

Contrarily, the second approach directly elicits also the cardinal part of the decision maker's preferences, however, only an approximate version of it. Specifically, this approximate version is obtained by eliciting, besides the rankings of the alternatives, also categories of preference strength. These categories are then utilized to construct a coarsened version R_2^c of the cardinal part R_2 of the underlying preference system. Again, we give conditions under which the elicited preference system is a sub-system of the decision maker's true one.

For both approaches, we investigate how efficiency can be improved by incorporating ranking data from previous elicitation rounds and/or expert knowledge. Two different approaches for improving the procedures are considered. On the one hand, the data is used to predict (and present) that pair of alternatives in the next elicitation step that reveals as much information as possible about the non-specified part of the preference system. On the other hand, the data is used for proposing the decision maker an 'educated guess' for a possible ranking, whenever two consequences are incomparable because of lacking information.

Finally, we demonstrate how the elicitation methods can be utilized to solve decision problems under severe uncertainty, while demanding the decision maker to specify exactly as much information as is needed for the specific decision problem under consideration. More precisely, we present algorithms that terminate the corresponding elicitation procedure as soon as it is possible to choose an optimal act from the underlying decision problem. In this way, the decision maker is guaranteed to obtain efficient decision support saving time and resources by not collecting unnecessary information.

References

[1] C. Jansen, G. Schollmeyer, and T. Augustin. Concepts for decision making under severe uncertainty with partial ordinal and partial cardinal preferences. *International Journal of Approximate Reasoning*, 98:112–131, 2018.

[2] D. Krantz, R. Luce, P. Suppes, and A. Tversky. *Foundations of Measurement. Volume I: Additive and Polynomial Representations*. Academic Press, San Diego and London, 1971.