Beyond Tree-Shaped Credal Sum-Product Networks*

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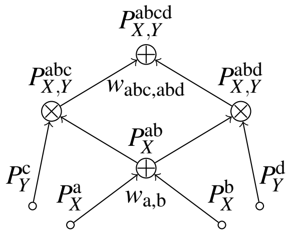
We present ongoing work that moves credal sum-product networks beyond the confines of tree-shaped graphs. The goal is to increase the number of cases where imprecision can be added to standard sum-product networks, while still retaining the possibility of efficient inference. This has been achieved with extensions to acyclic digraphs where the impact of imprecision on inference computations is isolated to parts of the network that are not the focus of the inference.

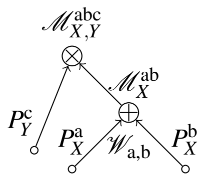
Context Probabilistic models on high-dimensional spaces are a natural choice for representing knowledge for artificial intelligence (AI) and machine learning (ML) applications, given their success for low-dimensional situations. An example relevant for the recognition of handwritten characters scanned into black-and-white raster images is the joint probability distribution of the random variables representing the images' pixel values.



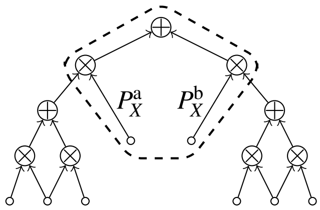
The search for efficient inference and decision making (e.g., classifying characters) has led the probabilistic AI and ML communities to joint distributions described by graphical representations (e.g., $P^{\mathrm{abcd}}_{X,Y}$ in the illustration on the right). A very successful such class of models are sum-product networks (SPNs, Poon and Domingos, 2011).

SPNs build a high-dimensional joint 'root' distribution from typically univariate 'leaf' distributions (P^{b}_{X}, P^{d}_{Y},...) by repeated application of 'sum' and (independent) 'product' operations (nodes $\oplus$ and $\otimes$ in the SPN's graph). This leads to a computational graph structure that allows for efficient computation of expectations of query functions that factorize over the random variables (X, Y), enabling practical applications such as character recognition. Sum operations correspond to nontrivial convex mixtures of probability distributions that are defined by some vector of weights $(w_{\mathrm{a,b}}, w_{\mathrm{abc,abd}})$.

Credal Sum-Product Networks Mauá et al. (2018) generalized SPNs to the imprecise-probabilistic context. They introduced imprecision by attaching sets of weight vectors (e.g., $\mathscr{W}_{\mathrm{a,b}}$ in the illustration on the right) to sum nodes, resulting in joint credal sets ($\mathscr{M}^{\mathrm{ab}}_{X}$, $\mathscr{M}^{\mathrm{abc}}_{X,Y}$). The product was generalized to the imprecise case by using the (sensitivity analysis-style) complete product. To ensure efficient calculation of lower and upper expectations, they restricted attention to SPNs with tree-shaped graphs and 'focused' query functions, i.e., those for which all factors are indicators except for the 'focus' variable (X here) whose factor is a difference of indicators. All query functions are products of functions over single variables.



We can relax the restriction to trees and focused query functions as long as the combination of these relaxations is compatible with efficient inference of lower and upper expectations. We have identified the following successful combinations:



1. A general (factorized) query function combined with a tree.
2. A general (factorized) query function where all factor functions are non-negative. This includes the 'likelihood' case of Mauá et al. (2018) as a subcase.
3. A focused query function combined with a graph for which no cycle edge is part of the paths from root to leafs describing the focus variable (illustration on the left).

References

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* This poster presents results of the ongoing Master's thesis project of Tijn Centen. His co-authors are actively involved in the supervision of this project, bringing ideas to the table. The authors are listed in alphabetic order. They thank the reviewer for their accurate and useful feedback.