Wasserstein Perturbations of Markovian Transition Probabilities

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When considering stochastic processes for the modeling of real world phenomena, a major issue is so-called model uncertainty or epistemic uncertainty. The latter refer to the impossibility of perfectly capturing information about the future in a single stochastic framework. The relevance of model uncertainty within a stochastic and nonstochastic setting is widely recognised within many fields such as statistics, operations research, finance, economic theory, and other related fields, and one typically differentiates between parametric and nonparametric uncertainty. Parametric uncertainty relates to the lack of information regarding certain parameters of a model while taking other model-specific assumptions as given. In a dynamic setting, the construction of consistent families of nonlinear transition semigroups has received a lot of attention in the past decades, and closely relates to the computation of price bounds in financial markets with incomplete information using methods from stochastic optimal control. On the other hand, nonparametric uncertainty refers to the impossibility of precisely capturing certain aspects of a model, such as independence or, more generally, the joint distributions of certain random variables – phenomena that are in general not implementable using a finite number of parameters. Although, in many cases, it is not straightforward how to mathematically quantify nonparametric uncertainty, one way to tackle such situations is to consider perturbations of a reference model using a certain notion of proximity, e.g., Wasserstein distances. In this talk, we follow this approach and consider nonparametric model uncertainty in terms of Wasserstein balls around a family of reference transition probabilities over finer and finer time periods, where the radius of the balls scales proportional to the time. By progressively making the partition of the time axis finer, we obtain as a limit a nonlinear semigroup satisfying a possibly fully nonlinear PDE in a viscosity sense. A remarkable observation is that the nonlinear transition operators arising from nonparametric uncertainty in a Wasserstein sense, in most cases, coincide with the ones related to parametric uncertainty. That is, in the limit, the Wasserstein distance is somewhat blind to actual nonparametric uncertainty, and only captures certain forms of parameter uncertainty. The talk is based on joint work with Sven Fuhrmann (University of Konstanz) and Michael Kupper (University of Konstanz).