Hierarchical Probabilistic Intervals *

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One of the basic problems in imprecise probabilities is to check if a set of assessments (upper and lower previsions) avoids sure loss and, if yes, to compute its natural extension [3]. Other interesting problems are computing the set of extreme probabilities or determining the conditional upper and lower previsions. In general, these are expensive computational problems for general previsions, but in some particular relevant cases, they can be efficiently solved. One the most important ones is the case of elementary probability intervals [1].

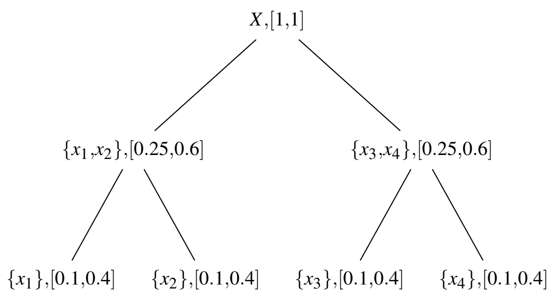
If X is a finite frame of discernment an elementary interval probabilistic specification is a set of probability intervals for each element $x \in X$, i.e. a family of intervals $[\underline{P}(\{x\}), \overline{P}(\{x\})], \quad x \in X$. In [1] simple arithmetic expressions were given for checking avoiding sure loss, coherence, and computing natural extension. Furthermore, it was also shown that its natural extension to the set of all the events $A \subseteq X$ was always an order-2 capacity [2] which also allows for simple expressions for conditional information and the development of an algorithm to compute the set of extreme probabilities.

In this work, we extend the results for elementary intervals to the more general setting of hierarchical interval specifications. A hierarchical interval specification is a family of intervals $[\underline{P}(A), \overline{P}(A)]$, for events $A \in \mathscr{A}$ where $\mathscr{A}$ is a family of subsets of X a family of subsets of X that can be structured as a tree T, with a set from $\mathscr{A}$ in each one of its nodes, and satisfying that the root is always X and that for each non-leaf node with set B, the sets associated with its children, $B_1 \ldots, B_k$ are a partition of B.

As an example, assume that $X = \{x_1, x_2, x_3, x_4\}$ and the specification given by the following sets of intervals:

$$\{x_i\} \to [0.1, 0.4], \; i = 1,2,3,4$$
$$\{x_1, x_2\} \to [0.25, 0.6], \; \{x_3, x_4\} \to [0.25, 0.6]$$
$$X \to [1,1]$$

The associated interval tree is the following one:



This work shows that the natural extension of this type of specifications to all the events is always an order-2 capacity. It also gives efficient algorithms for checking avoiding sure loss, checking coherence, and for computing natural extension, conditional upper and lower probabilities, or the extreme probabilities.

References

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[3] P. Walley. *Statistical Reasoning with Imprecise Probabilities*. Chapman and Hall, London, 1991.

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