Jensen-Like Inequalities for Imprecise Previsions

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We investigate how some well-known probability inequalities generalise in the realm of imprecise previsions. Two basic features of this problem are: (a) more than one inequality may correspond to a single probability inequality, and (b) the imprecise inequalities apply to previsions not necessarily coherent, but satisfying weaker consistency requirements.

As for (b), given a lower (upper) prevision $\underline{P}$ ($\overline{P}$) defined on a convenient set of gambles, the requirement that $\underline{P}$ ($\overline{P}$) is 2-coherent [4] is sufficient to derive a number of inequalities. For instance, assuming further that X is a non-negative gamble (and referring also to (a)), it is not difficult to show that two *Markov inequalities* apply:

$$\underline{P}(X \geq a) \leq \frac{\underline{P}(X)}{a}, \ \ \overline{P}(X \geq a) \leq \frac{\overline{P}(X)}{a}, \forall a > 0.$$

Extending Jensen's inequality is less immediate. While most previous work on this referred to generalisations of mathematical expectations via Choquet or other integrals [2, 3], the work in [1] focuses on functionals with rather general properties and can be extended to imprecise previsions with some further elaboration. The result is

Theorem (Jensen's inequalities) *Let X be a gamble, $\underline{P}$ a 2-coherent lower prevision and $\overline{P}$ its conjugate, $I \supset [\inf X, \sup X]$ an open interval, $\phi : I \to \mathbb{R}$ a convex function, $\psi : I \to \mathbb{R}$ a concave function. Then*

$$\underline{P}(\phi(X)) \geq \max\{\phi(\underline{P}(X)), \phi(\overline{P}(X))\}, \ \ \overline{P}(\psi(X)) \leq \min\{\psi(\underline{P}(X)), \psi(\overline{P}(X))\}.$$

As a follow-up of Jensen's inequalities we obtain an imprecise version of *Kolmogorov's inequality*: for any given s, t, $0 < s < t$,

$$[\overline{P}(|X|^s)]^{\frac{1}{s}} \leq [\underline{P}(|X|^t)]^{\frac{1}{t}}.$$

Further results (not reported here) concern improving Jensen's inequality under certain conditions, and the role of Jensen-like inequalities in evaluating imprecise even moments.

While what mentioned so far obtains with coherent previsions too, we show that extending Jensen's inequality is less manageable with other consistency notions, such as centered convexity. Technically, this depends on the positive homogeneity axiom $\mu(\lambda X) = \lambda\mu(X), \forall \lambda \geq 0$ ($\mu = \underline{P}$ or $\mu = \overline{P}$). This axiom applies with 2-monotonicity, but not with convexity. There is anyway no guarantee that 2-monotonicity performs better than convexity with generalisations of other probability inequalities.

References

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